Sulcal Pattern Mathching with the Wasserstein Distance

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We present the unified computational framework for modeling the sulcal patterns of human brain obtained from MRI. These patterns are topologically different across subjects making the pattern matching a challenge. We use the Wasserstein distance in aligning align the sulcal patterns nonlinearly. The codes are provided in https://github.com/laplcebeltrami/sulcaltree.

Sulcal pattern projection

We used T1-weighted MRI of 456 subjects (age-matched 274 females and 182 males) in the Human Connectome Project [2]. The TRACE algorithm is used for automatic sulcal curve extraction from surface meshes and project them onto a plane [4]. The sulcal pattern $f(x, y)$ is generated by assigning value 1 to sulcal curves and 0 otherwise.

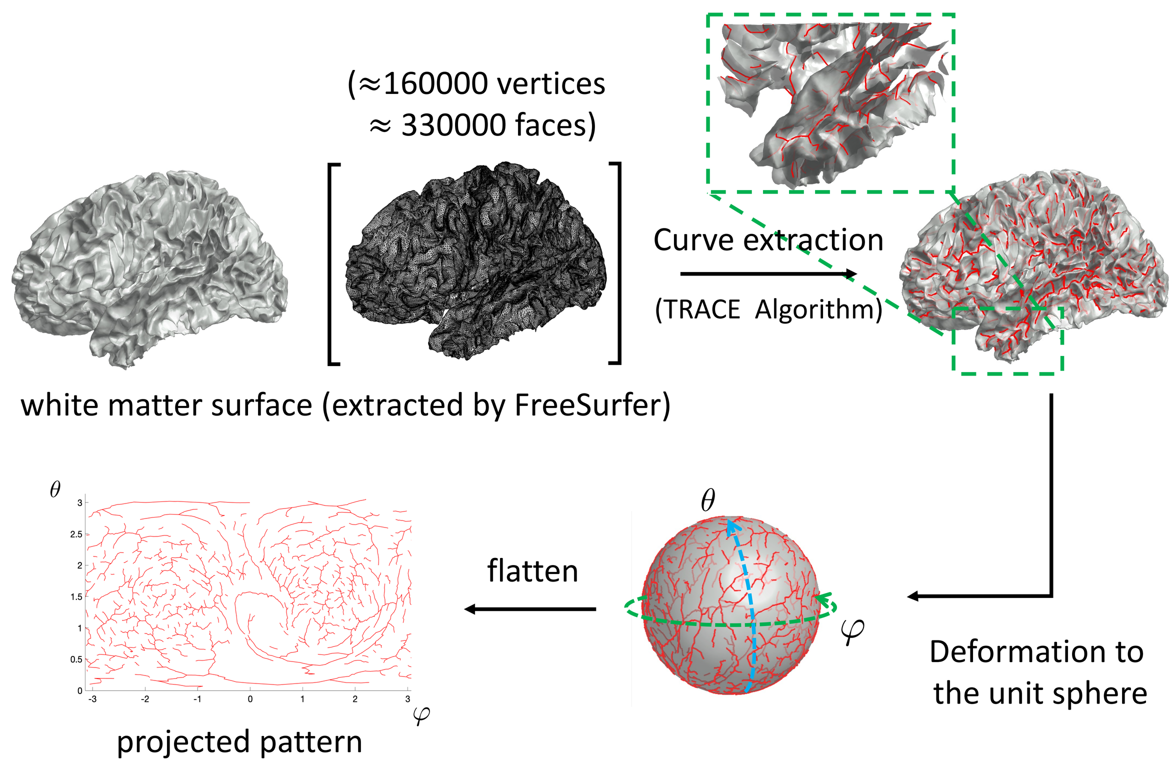

Figure 1: Schematic of the sulcal curve extraction and projection.

Heat Kernel Smoothing

Heat kernel smoothing was performed on the projected sulcal pattern by solving

$$\frac{\partial}{\partial\sigma}u(x,y,\sigma) = \frac{\partial^2}{\partial^2 x}u(x,y,\sigma) + \frac{\partial^2}{\partial^2 y}u(x,y,\sigma)$$

with the initial condition $u(x, y, \sigma = 0) = f(x, y)$ and the periodic boundary conditions in Figure 2. The solution is given as the weighted Fourier series

$$\bar{\mu}(x,y) = \sum_{j=0}^{m}\sum_{k=0}^{n} e^{-\lambda_{jk}\sigma}\left[A_{jk}\phi^1_{jk} + B_{jk}\phi^2_{jk}\right].$$

The eigenvalues $\lambda_{jk} = j^2 + k^2$ and the eigenfunctions corresponding to Laplacian $\frac{\partial^2}{\partial^2 x} + \frac{\partial^2}{\partial^2 y}$ are

$$\phi^1_{jk} = \frac{2\cos(jx)\sin(ky)}{\pi^2(1+\delta_{j0})}, \phi^2_{jk} = \frac{2\sin(jx)\sin(ky)}{\pi^2}$$

with $\delta_{j0} = 1$ if $j = 0$ and 0 otherwise. A_{jk} and B_{jk} are estimated in the least square fashion [2].

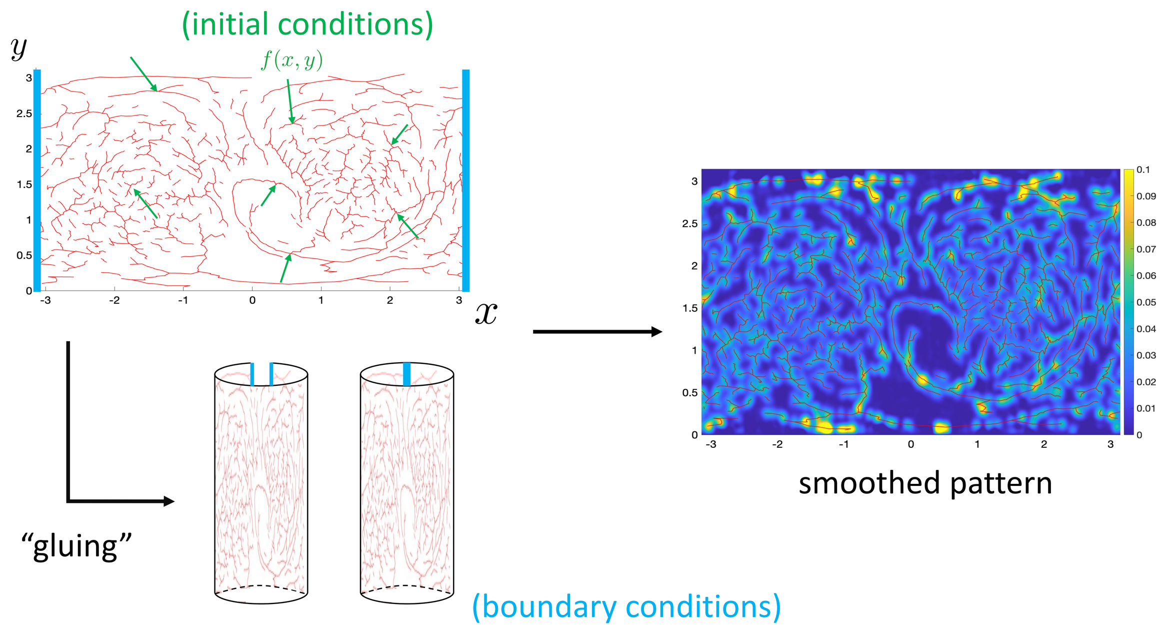

Figure 2: Heat kernel smoothing of projected pattern with $\sigma = 0.001$.

Matching via Wasserstein distance

Let f_1 and f_2 be the two probability density defined on Ω_p and Ω_q. The Kantorovich's formulation for Wasserstein distance is defined as

$$D_W(f_1, f_2) = \inf_{\pi}\left(\int\int \|p-q\|^2\pi(p,q)dp\, dq\right)^{\frac{1}{2}}$$

over all possible joint density π with marginals f_1 and f_2 [1]. In the dual formulation, we solve

$$\inf_{\varphi+\psi\geq\langle p,q\rangle}\left\{\int_{\Omega_p}\varphi(p)f_1(p)dp + \int_{\Omega_q}\psi(q)f_2(q)dq\right\}.$$

The unique solution is given by

$$\psi = \max_p\left(p^\top q - \varphi(p)\right).$$

The corresponding objective function is written as $L(\varphi)$ given by [1]

$$L'(\varphi) = f_1 - (f_2 \circ \nabla\varphi)\det(H_\varphi)$$

with deformation $\nabla\varphi$. The gradient descent is performed in solving for φ:

$$\varphi_{n+1} = \varphi_n - L'(\varphi_n).$$

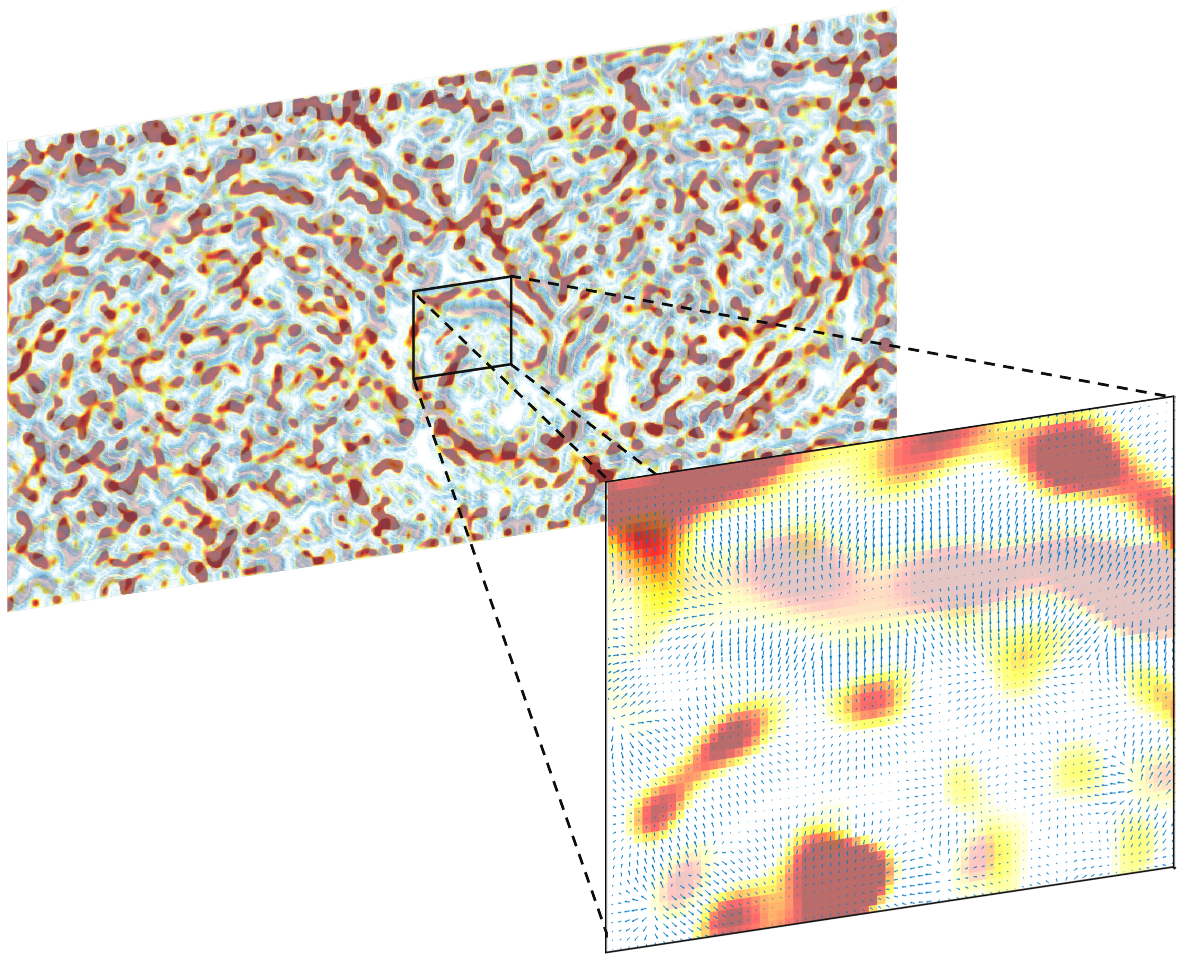

Figure 3: Subject 1 (lower transparency) superimposed on top of subject 2 (higher transparency). The arrows are the displacement field of matching subject 1 to subject 2.

Wasserstein distance after diffusion

Suppose f_1 and f_2 are two empirical distributions on points p_i and q_i that define sulcal curves

$$f_1(p) = \frac{1}{n}\sum_{i=1}^{n}\delta(p-p_i), f_2(q) = \frac{1}{n}\sum_{i=1}^{n}\delta(q-q_i).$$

The heat kernel smoothing of f_1 and f_2 is given by

$$\tilde{f}_1(p) = \frac{1}{n}\sum_{i=1}^{n}K_\sigma(p,p_i), \tilde{f}_2(q) = \frac{1}{n}\sum_{i=1}^{n}K_\sigma(q,q_i).$$

Theorem *For restricted Wasserstein distance $D_{W'}$ on Gaussian distributions, we have*

$$D_{W'}(f_1, f_2) = D_W(f_1, f_2).$$

Thus, the Wasserstein distance is reduced after heat kernel smoothing

$$D_W(\tilde{f}_1, \tilde{f}_2) \leq D_W(f_1, f_2).$$

Sexual dimorphism

The method is subsequently used to determine the sulcal pattern differences between females and males. The registration pipeline is summarized in Figure 4. Our pipeline significantly reduces the sulcal pattern variability compared to FreeSurfer's folding-based surface registration method. The intersubject variability of the smoothed original sulcal patterns (left) and deformed patterns (right). The statistical variability after registration is reduced by 96.29%.

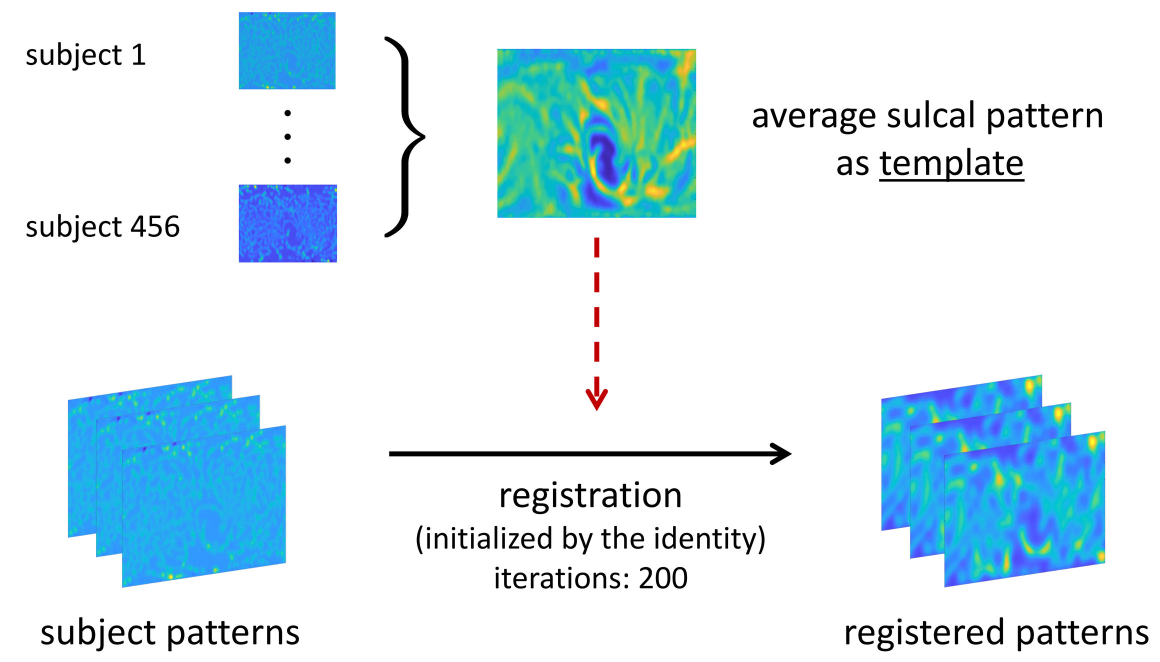

Figure 4: Schematic of the Wasserstein-based registration.

The two sample t-statistics (female-male) were constructed and thresholded that correspond to the p-value of 0.05 after multiple comparisons correction through the permutation test (half millions). The negative t-statistics in most of the brain regions indicates the presence of less sulci for females, which is related to the previous finding [3].

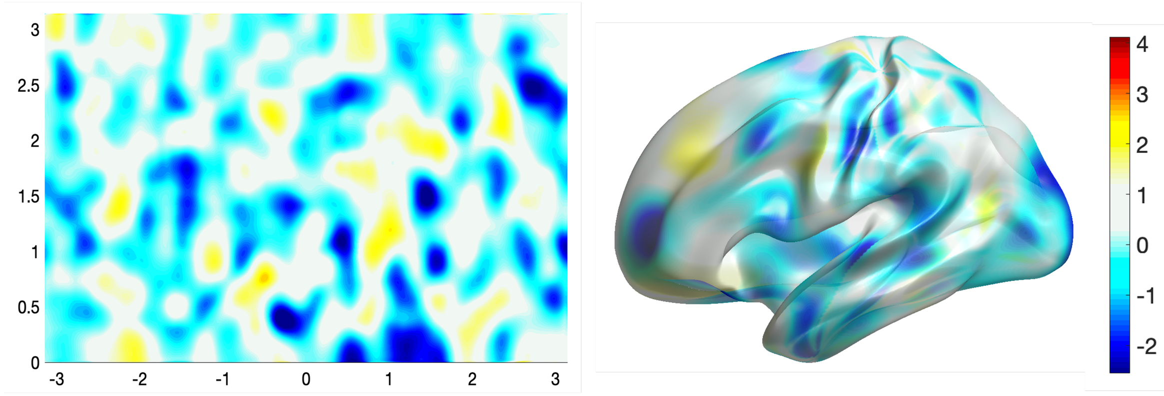

Figure 5: t-statistics map of the localized sulcal pattern differences. Values are thresholded in $[-2.84, 4.12]$ corresponding to the corrected p-value of 0.05.

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References

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